

## AP Calculus BC – AP Exam Review Chart

When you see this...		Do this...
1.	Find the zeros	SET $f(x) = 0$ , SOLVE BY FACTORING OR USE YOUR CALCULATOR.
2.	Find where $f(x) = g(x)$	SOLVE $f(x) - g(x) = 0$ .
3.	Find the equation of the line tangent to $f(x)$ at $x = a$	$y - f(a) = f'(a)(x - a)$
4.	Find the equation of the line normal to $f(x)$ at $x = a$	$y - f(a) = \frac{-1}{f'(a)}(x - a)$
5.	Use the equation of the tangent line to $f(x)$ at $x = a$ to approximate $f(b)$	$f(b) \approx f(a) + f'(a)(b - a)$
6.	$\frac{d}{dx}(f(x)g(x)) =$	$f(x)g'(x) + g(x)f'(x)$ PRODUCT RULE
7.	$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) =$	$\frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ QUOTIENT RULE
8.	$\frac{d}{dx}(f(g(x))) =$	$f'(g(x)) \cdot g'(x)$ CHAIN RULE
9.	Find where the tangent line to $f(x)$ is horizontal/vertical	HORIZONTAL: $dy/dx = 0$ VERTICAL: $dy/dx$ IS UNDEFINED
10.	Find the interval(s) where $f(x)$ is increasing/decreasing	INCR: $f'(x) > 0$ DECR: $f'(x) < 0$
11.	Find the interval(s) where the slope of $f(x)$ is increasing/decreasing	INC: $f''(x) > 0$ f CONCAVE UP DEC: $f''(x) < 0$ f CONCAVE DOWN
12.	Find the interval(s) where $f(x)$ is concave up/down	UP: $f''(x) > 0$ DOWN: $f''(x) < 0$
13.	Find the maximum/minimum values of $f(x)$ on $[a,b]$	USE EVT: COMPARE $f(a)$ AND $f(b)$ TO FUNCTION AT CRIT. PTS. ANSWER IS y-VALUE.
14.	Find critical points	FIND WHERE $f'(x) = 0$ OR WHERE $f'(x)$ DOES NOT EXIST (IN DOMAIN OF f)
15.	Find and verify relative extrema – 1 <sup>st</sup> deriv test	VERIFY CRIT. PT. AT $x = a$ . SHOW $f'$ CHANGES FROM + TO - (MAX) OR - TO + (MIN) AT a
16.	Find and verify relative extrema – 2 <sup>nd</sup> deriv test	VERIFY A CRIT PT AT $x = a$ . SHOW $f''(a) > 0$ FOR A MIN; $f''(a) < 0$ FOR A MAX.
17.	Find and verify inflection points	SHOW THAT $f''(x)$ CHANGES SIGN AT $x = a$ . $f''(a) = 0$ OR IS UNDEFINED.
18.	Show that $\lim_{x \rightarrow a} f(x)$ exists	SHOW $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$

19.	Show that $f(x)$ is continuous at $x=a$	SHOW $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$
20.	Show that $f(x)$ is differentiable (or not) at a given point	SHOW $\lim_{x \rightarrow a^-} f'(x) = \lim_{x \rightarrow a^+} f'(x)$
21.	Find vertical asymptotes of $f(x)$	FIND VALUES WHERE $f(x)$ IS UNDEFINED (ASSUMING YOU CHECKED FOR HOLES)
22.	Find horizontal asymptotes of $f(x)$	$\lim_{x \rightarrow \pm\infty} f(x)$
23.	Find the average rate of change of $f(x)$ on $[a,b]$	$\frac{f(b) - f(a)}{b-a} = \frac{1}{b-a} \int_a^b f'(x) dx$
24.	Find the instantaneous rate of change of $f(x)$ at $x=a$	$f'(a)$
25.	Find the average value of $f(x)$ on $[a,b]$	$\frac{1}{b-a} \int_a^b f(x) dx$
26.	Show that a piecewise function is continuous or differentiable at a point $a$ (where the function splits)	CMTIN: SHOW $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$ DIFF: SHOW $\lim_{x \rightarrow a^-} f'(x) = \lim_{x \rightarrow a^+} f'(x)$
27.	Given a position function, find the velocity and acceleration functions	$v(t) = s'(t)$ $a(t) = v'(t) = s''(t)$
28.	Find the displacement of a moving particle on the interval $[a,b]$	$\int_a^b s(t) dt$
29.	Find $x(t_2)$ given $x(t_1)$ and $v(t)$	$x(t_2) = x(t_1) + \int_{t_1}^{t_2} v(t) dt$
30.	Find the speed of the particle at a given value of $t$	$ v(t) $
31.	Find the total distance traveled on $[a,b]$	$\int_a^b  s(t)  dt$
32.	Find the average velocity of the particle on $[a,b]$	$\frac{s(b) - s(a)}{b-a}$ OR $\frac{1}{b-a} \int_a^b v(t) dt$
33.	Determine whether an object is speeding up/slowing down	SPEEDING UP: $v(t), a(t)$ SAME SIGN SLOWING DOWN: $v(t), a(t)$ OPPOSITE SIGN
34.	Show that the Mean Value Theorem holds (or does not hold) on $[a,b]$ for a given function	VERIFY $f(x)$ IS CNT. AND DIFF. ON $[a,b]$ .
35.	Find the domain of $f(x)$	FIND VALUES OF $x$ SUCH THAT $f(x)$ EXISTS.
36.	Find the range of $f(x)$	FIND POSSIBLE VALUES OF $f(x)$ ON THE GIVEN DOMAIN.
37.	Find $f'(x)$ using the definition of the derivative	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

38.	Find the derivative of the inverse of $f(x)$ at $x = a$	$\frac{d}{dx} f^{-1}(x) \Big _{x=a} = \frac{1}{f'(f^{-1}(x))} \Big _{x=a}$
39.	Given that the rate of change of $y$ is proportional to $y$ , find an expression for $y$	$\frac{dy/dt}{y} = k \Rightarrow \frac{dy}{dt} = ky \Rightarrow y = y_0 e^{kt}$ EXPONENTIAL GROWTH/DECAY
40.	Find the line $x = c$ that divides the area under $f(x)$ on $[a,b]$ into two equal areas	$\int_a^c f(x) dx = \frac{1}{2} \int_a^b f(x) dx$
41.	$\int_a^b f'(x) dx =$	$f(x) \Big _a^b = f(b) - f(a)$
42.	$\frac{d}{dx} \int_a^x f(t) dt =$	$f(x)$
43.	$\frac{d}{dx} \int_a^{g(x)} f(t) dt =$	$f(g(x)) \cdot g'(x)$
44.	$\frac{d}{dx} \int_{h(x)}^{g(x)} f(t) dt =$	$f(g(x)) \cdot g'(x) - f(h(x)) \cdot h'(x)$
45.	Approximate $\int_a^b f(x) dx$ using 4 subintervals* and the given method	<p>a) <math>2(1+13+16+5) = 70</math>      b) <math>2(13+16+5+3) = 74</math>      c) <math>4(13+5) = 72</math>      d) <math>\frac{1}{2} \cdot 2(1+2(13)+2(16)+2(5)+3) = 72</math></p> <p> <math display="block">\begin{array}{ c c c c c c } \hline x &amp; 1 &amp; 3 &amp; 5 &amp; 7 &amp; 9 \\ \hline f(x) &amp; 1 &amp; 13 &amp; 16 &amp; 5 &amp; 3 \\ \hline \end{array}</math>         a. LRAM        b. RRAM        c. MRAM (*2 subintervals)        d. TRAP     </p>
46.	Given the table above, approximate $f'(3)$	$f'(3) \approx \frac{f(5)-f(1)}{5-1} = \frac{16-1}{4} = \frac{15}{4}$
47.	Find the particular solution $y=f(x)$ to $\frac{dy}{dx} = \dots$	SEPARATE... THEN INTEGRATE!
48.	Given a differential equation $\frac{dy}{dx} = f(x, y)$ , draw a slope field and a particular solution through a given point	FIND SLOPE AT ANY PT $(x_n, y_n)$ BY PLUGGING INTO $\frac{dy}{dx} = f(x, y)$ . DRAW SEGMENTS AT $(x_n, y_n)$ w/ SLOPE - MAKE SURE YOU HAVE ZERO SLOPES!
49.	Given a differential equation $\frac{dy}{dx} = f(x, y)$ , show that $y=f(x)$ is a solution.	PLUG $y$ AND $y'$ INTO THE DIFF. EQUATION; SHOW TWO SIDES OF EQUATION ARE = CORRECT!
50.	Euler's Method: If $\frac{dy}{dx} = f(x, y)$ and $(x_0, y_0)$ is a point on the solution curve, then $y_1 =$	$y_1 = y_0 + f(x_0, y_0) \Delta x$ / $y_{n+1} = y_n + f(x_n, y_n) \Delta x$

$\Delta x = \text{STEP SIZE}$

51.	Find the area contained by two functions (with respect to $x$ )	$\int_{x_1}^{x_2} \text{TOP FKN - BOTTOM FKN } dx$
52.	Find the area contained by two functions (with respect to $y$ )	$\int_{y_1}^{y_2} \text{RIGHT FKN - LEFT FKN } dy$
53.	Find the volume of a solid with known cross-sectional area $A(x)$ whose base is the area under $f(x)$ on $[a,b]$	$\int_a^b A(x) dx$
54.	Find the volume if the area under $f(x)$ and above the $x$ -axis from $[a,b]$ is rotated about the:	a) $\pi \int_a^b [f(x)]^2 dx$ b) $\pi \int_a^b [f(x)]^2 - c^2 dx$ IF $f(x) > c$ $\pi \int_a^b c^2 - [f(x)]^2 dx$ IF $c > f(x)$
55.	Find the volume if the area between $f(x)$ and $g(x)$ is rotated about the:  a. $x$ -axis b. line $x = c$	a) $\pi \int_a^b [f(x)]^2 - [g(x)]^2 dx$ IF $f(x) > g(x)$ b) $\pi \int_a^b [f(x)-c]^2 - [g(x)-c]^2 dx$ IF $f(x) > g(x) > c$
56.	Repeat #54 and #55 with functions with respect to $y$ and rotating about the:  a. $y$ -axis b. line $y = c$	(24) a) $\int_{y_1}^{y_2} [f(y)]^2 dy$ (25) a) $\int_{y_1}^{y_2} [f(y)]^2 - [g(y)]^2 dy$ b) $\int_{y_1}^{y_2} [f(y)]^2 - c^2 dy$ b) $\int_{y_1}^{y_2} [f(y)-c]^2 - [g(y)-c]^2 dy$ $f(y) > c$ $f(y) > g(y) > c$
57.	Find the length of a curve (function mode)	$\int_{x_1}^{x_2} \sqrt{1 + [f'(x)]^2} dx$
58.	Find $f(b)$ given $f'(x)$ and $f(a)$	$f(b) = f(a) + \int_a^b f'(x) dx$
59.	Given a graph of $f'(x)$ , determine where $f(x)$ is:  a. Increasing/decreasing b. Concave up/down  Also determine relative extrema and points of inflection.	a) $f$ INCR IF $f' > 0 \Rightarrow$ FIND WHERE GRAPH OF $f'$ IS POSITIVE. $f$ DECR IF $f' < 0 \Rightarrow$ FIND WHERE GRAPH OF $f'$ IS NEGATIVE b) $f$ CCT UP IF $f'' > 0 \Rightarrow$ FIND WHERE GRAPH OF $f'$ IS INCREASING $f$ CCT IF $f'' < 0 \Rightarrow$ GRAPH OF $f'$ DECREASING.
60.	Integration by Parts: $\int u dv = uv - \int v du$ LIPET = ORDER TO PICK $u$	EXTREMA: GRAPH OF $f'$ CHANGES + TO - OR - TO + P.O.I.: GRAPH OF $f'$ CHANGES INCR TO DECR OR
61.	Partial Fractions (cover up)  a. For what types of functions can it be used? b. What must be true about the denominator?	VICE VERSA a) RATIONAL FUNCTIONS, CAN FACTOR DENOM. b) DEGREE OF DENOM > DEGREE NUM. (OTHERWISE, DO LONG DIVISION FIRST)

		$\infty - \infty \Rightarrow$ COMBINE TERMS
62.	L'Hopital's Rule: for what indeterminate forms can it be used?	$\frac{0}{0} \text{ or } \frac{\infty}{\infty} \Rightarrow$ REWRITE AS $0/0$ OR $\infty/\infty$ $0^0, 1^\infty, \infty^0 \Rightarrow$ TAKE LN OF BOTH SIDES
63.	Improper Integrals: what makes an integral improper?	ONE OF LIMITS IS $\pm \infty$ ONE OF LIMITS IS VERTICAL ASYMPTOTE
64.	$\frac{dP}{dt} = \frac{k}{M} P(M-P)$  a. What does $M$ stand for? b. What is $\lim_{t \rightarrow \infty} P(t)$ c. When is the population growing fastest?	a) $M =$ CARRYING CAPACITY b) $\lim_{t \rightarrow \infty} P(t) = M$ c) WHEN $P = M/2$
65.	Vectors: if $r(t) = \langle x(t), y(t) \rangle$ , then:  a. $v(t) =$ b. $a(t) =$ c. Speed = d. Total Distance traveled (arc length) on $[a, b] =$ e. $\frac{dy}{dx} =$ f. Object at rest if...	a) $\langle x'(t), y'(t) \rangle$ b) $\langle x''(t), y''(t) \rangle$ c) $ v  = \sqrt{(x'(t))^2 + (y'(t))^2}$ d) $\int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} dt$ e) $\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$ f) $x'(t)$ AND $y'(t)$ BOTH = 0 FOR SAME $t$
66.	Find the area contained by a polar curve	$\frac{1}{2} \int_{\theta_1}^{\theta_2} r^2 d\theta$
67.	Converting Cartesian to polar:  a. $x = r \cos \theta$ b. $y = r \sin \theta$	$x^2 + y^2 = r^2$ $\theta = \tan^{-1}(y/x)$
68.	Slope in polar: $\frac{dy}{dx} = \frac{\frac{dr}{d\theta}(r \sin \theta)}{\frac{dr}{d\theta}(r \cos \theta)} = \frac{r \sin \theta}{r \cos \theta + \frac{dr}{d\theta} \sin \theta}$	$= \frac{r \cos \theta + \frac{dr}{d\theta} \sin \theta}{-r \sin \theta + \frac{dr}{d\theta} \cos \theta}$
69.	Find the length of a polar curve	$\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$
70.	Determine the convergence/divergence of a series using:  a. Divergence test b. Integral test c. P-series test $\sum_{n=1}^{\infty} \frac{1}{n^p}$ d. Geometric series test $\sum_{n=1}^{\infty} ar^{n-1}$ e. Ratio test	a) IF $\lim_{n \rightarrow \infty} a_n \neq 0 \Rightarrow$ SERIES DIV. b) $\int_a^{\infty} a(x) dx$ AND $\sum_{n=1}^{\infty} a_n$ BOTH CONVERGE OR BOTH DIVERGE c) $p > 1 \Rightarrow$ CONV. $0 < p \leq 1 \Rightarrow$ DIVERGE d) CONV. IF $ r  < 1$ SUM = $\frac{a}{1-r}$ e) $\rho = \lim_{n \rightarrow \infty} \left  \frac{a_{n+1}}{a_n} \right  < 1 \Rightarrow$ CONV.  $> 1 \Rightarrow$ DIV $= 1 \Rightarrow$ INCONCLUSIVE

71.	Find the interval/radius of convergence of a series	DO RATIO TEST FOR ABSOLUTE CONVERGENCE $\rho = \lim_{n \rightarrow \infty}  a_{n+1}/a_n  < 1$
72.	Write the Taylor series about $x = a$	$f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \dots$
73.	Write the Maclaurin series for  a. $\sin x$ b. $\cos x$ c. $e^x$ d. $\frac{1}{1-x}$ e. $\frac{1}{1+x}$	a) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \dots$ b) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + \frac{(-1)^n x^{2n}}{(2n)!} + \dots$ c) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$ d) $1 + x + x^2 + x^3 + \dots + x^n + \dots$ e) $1 - x + x^2 - x^3 + \dots + (-1)^n x^n + \dots$
74.	Write a series for each of the following (using known series):  a. $\frac{\cos(3x)+1}{x}$ b. $\frac{e^{-x^2}}{x}$	a) $\left( 1 - \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!} - \dots + \frac{(-1)^n (3x)^{2n}}{(2n)!} \right) + 1$  b) $\frac{1}{x} \left( 1 - x^2 + \frac{x^2}{2!} + \frac{(-x^2)^3}{3!} + \dots + \frac{(-x^2)^n}{n!} + \dots \right)$ $\frac{1}{x} - x + \frac{x^3}{2!} - \frac{x^5}{3!} + \dots + \frac{(-1)^n (x^{2n-1})}{n!} + \dots$
75.	If $f(x) = 2 + 6x + 18x^2 + \dots$ , find $f\left(\frac{1}{6}\right)$	GEOMETRIC WITH $r = 3x \Rightarrow \text{sum} = \frac{2}{1-3x}$ $f(1/6) = \frac{2}{1-3(1/6)} = \frac{2}{1/2} = 4$
76.	Suppose $f^{(n)}(a) = \frac{(n+1)n!}{2^n}$ for $n \geq 1$ and $f(a) = 2$ . Write the first four terms and the general term of the Taylor series for $f(x)$ about $x = a$ .	$\begin{aligned} f(a) &= 2 \\ f'(a) &= \frac{2 \cdot 1!}{2^1} = 1 \\ f''(a) &= \frac{3 \cdot 2!}{2^2} \\ f'''(a) &= \frac{4 \cdot 3!}{2^3} \end{aligned} \quad \left. \begin{aligned} 2 + (x-a) + \frac{3}{2^2}(xa)^2 + \frac{4}{2^3}(xa)^3 + \\ + \frac{(n+1)(x-a)^n}{2^n} + \dots \end{aligned} \right.$
77.	Let $S_4$ be the sum of the first 4 terms of a converging alternating series that approximates $f(x)$ . Approximate $ f(x) - S_4 $	$ \text{error}  \leq  a_5 $
78.	What are the properties of a series that guarantee that the error in approximating $f(x)$ using $S_n$ is less than or equal to $a_{n+1}$ ?	ABSOLUTE VALUE OF TERMS ARE DECREASING TO ZERO.
79.	Given a Taylor series, find the Lagrange form of the remainder for the 4 <sup>th</sup> term	$\left  \frac{M}{5!} (x-a)^5 \right  \quad \text{WHERE } M = \text{MAX VALUE}$ OF $f^{(5)}(x)$ ON $(x-a, x+a)$